Mathematics, more than the fundamental language and underlying analytical structure of science and technology, is a formal way of thinking—an art that ties together the abstract structure of reason and the formal development of the logic that defines the scientific method. From the study of just how arguments and theories are formed in language and technology to the framework of quantitative and qualitative models of the natural and social sciences, mathematics is based upon the development of precise expressions, logical arguments, and the search for and exposure of pattern and structure.

The undergraduate program in the Department of Mathematics is intended both for students interested in attaining the proper preparation for graduate study and research in pure mathematics, and for students interested in using mathematics to define, properly pose, and solve problems in the sciences, engineering, and other areas. With either purpose, the focus of the program is to help those who wish to understand further the logical content, geometric meaning, and abstract reasoning of mathematics itself. A flexible program involving a broad selection of courses is a department tradition. The program begins by introducing students to the basics of algebra and mathematical analysis and then gives them the choice of exploring topics in theoretical mathematics or studying applications to physics, economics, engineering, computer science, probability, statistics, or mechanics.

The graduate program is designed primarily to prepare students for research and teaching in mathematics. It is naturally centered around the research areas of the faculty, which include algebraic geometry, algebraic number theory, differential geometry, partial differential equations, topology, several complex variables, algebraic groups, and representation theory. The program can be supplemented in applied directions by courses in theoretical physics, computer science, mechanics, probability, and statistics offered in other departments of the Krieger School of Arts and Sciences and in the Department of Applied Mathematics in the Whiting School of Engineering.

Facilities

The Mathematics Department resides in Krieger Hall on the Keyser Quad of Homewood. Adjacent to Krieger Hall, The University’s Milton S. Eisenhower Library has an unusually extensive collection of mathematics literature, including all the major research journals, almost all of which are accessible electronically. The stacks are open to students. The department also has a useful reference library, the Philip Hartman Library. Graduate students share departmental offices, and study space can also be reserved in the university library. Graduate students may access the department’s Linux and Windows servers, as well as computers in graduate student offices. The department also hosts numerous research seminars, special lectures, and conferences throughout the academic year.

Math Course Placement and Sequencing for All Students

There are different versions of Calculus I and II offered by the Mathematics Departments and students should select their first course in math at JHU based on their major intentions and placement. Students intending to major in mathematics, the physical sciences, or engineering are encouraged to begin with the AS.110.108 Calculus I - AS.110.109 Calculus II (For Physical Sciences and Engineering) sequence or AS.110.113 Honors Single Variable Calculus. Students majoring in other subjects may wish to take the AS.110.106 Calculus I (Biology and Social Sciences) - AS.110.107 Calculus II (For Biological and Social Science) sequence which relates the methods of calculus to the biological and social sciences. A one-semester pre-calculus course (AS.110.105 Introduction to Calculus) is offered for students who could benefit from additional preparation in the basic tools (algebra and trigonometry) used in calculus.

Entering students may receive course credit for Calculus I or Calculus I and II on the basis of the College Board Advanced Placement (AP) or International Baccalaureate (IB) exams (http://e-catalog.jhu.edu/undergrad-students/academic-policies/external-credit/#examcredittext). All students, regardless of completion of advanced placement exams previously, must take a departmental placement exam to determine their appropriate first course in mathematics. Additional placement information can be found here (http://www.mathematics.jhu.edu/new/undergrad/advising_placement.htm).

After completing a Calculus II course, the courses AS.110.201 Linear Algebra, AS.110.202 Calculus III, or AS.110.302 Diff Equations/Applic may be taken in any order. The department offers honors courses in both AS.110.212 Honors Linear Algebra and AS.110.211 Honors Multivariable Calculus.

Requirements for the B.A. Degree

In addition to the Requirements for a Bachelor's Degree (http://e-catalog.jhu.edu/undergrad-students/academic-policies/requirements-for-a-bachelors-degree), a candidate for the bachelor's degree in mathematics is required to have completed the major requirements listed below. All courses used to meet these requirements must be completed with a grade of C- or better and may not be taken satisfactory/unsatisfactory.

<table>
<thead>
<tr>
<th>Course</th>
<th>Credits</th>
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<tbody>
<tr>
<td>AS.110.106 Calculus I (Biology and Social Sciences)</td>
<td>4</td>
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<tr>
<td>or AS.110.108 Calculus I</td>
<td></td>
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<tr>
<td>or AS.110.113 Honors Single Variable Calculus</td>
<td></td>
</tr>
<tr>
<td>AS.110.109 Calculus II (For Physical Sciences and Engineering)</td>
<td>4.00</td>
</tr>
<tr>
<td>AS.110.202 Calculus III</td>
<td>4</td>
</tr>
<tr>
<td>or AS.110.211 Honors Multivariable Calculus</td>
<td></td>
</tr>
<tr>
<td>AS.110.201 Linear Algebra</td>
<td>4</td>
</tr>
<tr>
<td>or AS.110.212 Honors Linear Algebra</td>
<td></td>
</tr>
<tr>
<td>AS.110.401 Advanced Algebra I</td>
<td>4.00</td>
</tr>
<tr>
<td>AS.110.304 Elementary Number Theory</td>
<td>4</td>
</tr>
<tr>
<td>or AS.110.402 Advanced Algebra II</td>
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</tr>
<tr>
<td>AS.110.405 Analysis I</td>
<td>4</td>
</tr>
<tr>
<td>or AS.110.415 Honors Analysis I</td>
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</tbody>
</table>

One additional course selected from:

<table>
<thead>
<tr>
<th>Course</th>
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<tbody>
<tr>
<td>AS.110.311 Methods of Complex Analysis</td>
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<tr>
<td>or AS.110.406 Analysis II</td>
<td></td>
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<tr>
<td>or AS.110.412 Introduction To Topology</td>
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<td>or AS.110.413 Honors Analysis II</td>
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<td>or AS.110.414 Partial Diff Equations</td>
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<td>or AS.110.420 Dynamical Systems</td>
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<tr>
<td>or AS.110.421 Introduction Calculus of Variations</td>
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</tbody>
</table>
or AS.110.439 Introduction To Differential Geometry
or AS.110.443 Fourier Analysis
One 300-level or higher math course
Two courses in any one of the approved applications of mathematics or other courses approved by the Director of Undergraduate Studies

Total Credits
44

* Majors are encouraged but not required to take honors variant.
** See table below for approved application courses.

Approved courses in areas of application:

Physics
AS.171.204 Classical Mechanics II 4
AS.171.301 Electromagnetic Theory II 4
AS.171.303 Quantum Mechanics I 4
AS.171.304 Quantum Mechanics II 4
AS.171.312 Statistical Physics/Thermodynamics 4

Chemistry
AS.030.302 Physical Chemistry II 3
AS.030.345 Chemical Applications of Group Theory 3
AS.030.453 Intermediate Quantum Chemistry 3

Economics
AS.180.301 Microeconomic Theory 4
AS.180.302 Macroeconomic Theory 4

Computer Science
EN.600.435 Artificial Intelligence 3
EN.600.463 Algorithms I 3
EN.600.464 Randomized and Big Data Algorithms 3
EN.600.488 Foundations of Computational Biology & Bioinformatics II 3

Sample Program of Study
The following chart is one example of how a student might progress through the mathematics major. As potential math majors enter JHU with a wide range of prior math abilities, students should begin courses at their current level of knowledge.

Freshman
Fall
AS.110.108 Calculus I 4

Sophomore
Fall
AS.110.202 or Calculus III 4
AS.110.302 Diff Equations/ Applic (or other 300+ level elective) 4

Junior
Fall
AS.110.401 Advanced Algebra I 4
AS.110.402 or Advanced Algebra II 4
Math application course 3-4.5

Senior
Fall
AS.110.405 or Analysis I 4
AS.110.406 or Analysis II (or additional “analysis” course from list) 4
Math applications course 3-4.5

Total Credits: 42-45

Requirements for a Minor in Mathematics
All courses used to meet the mathematics minor requirements must be completed with a grade of C- or better and may not be taken satisfactory/unsatisfactory. One course in the Applied Mathematics and Statistics Department (at the 300-level or above) may be substituted for one of the elective courses for the minor.

AS.110.106 Calculus I (Biological and Social Sciences) 4
or AS.110.108 Calculus I 4
AS.110.107 Calculus II (For Biological and Social Science) 4
or AS.110.109 Calculus II (For Physical Sciences and Engineering) 4
or AS.110.113 Honors Single Variable Calculus 4
AS.110.202 Calculus III 4
One 200-level or above math course (excluding AS.110.202) 4
Three 300-level or above math courses 12

Total Credits 28

Honors Program in Mathematics
As a general guideline, departmental honors are awarded to recipients of the B.A. degree who have completed AS.110.311 Methods of Complex Analysis, as well as AS.110.401 Advanced Algebra I, AS.110.415 Honors Analysis I-AS.110.416 Honors Analysis II, and one more course at the 400-level or above with at least a 3.6 average in these six courses.

J.J. Sylvester Prize
The J.J. Sylvester Prize in Mathematics, which carries a cash award, is given each year to the one of two top-performing graduating seniors majoring in mathematics for outstanding achievement.

The B.A./M.A. Program
By applying the same courses simultaneously toward the requirements for the B.A. and M.A. degrees, an advanced student can qualify for both degrees in four years. Admission to the program is by the standard graduate application form, which should be completed in the junior year. At least a 3.0 average is required in the 400-level mathematics courses taken while resident at the university. Students may contact the graduate program assistant for further information.
Undergraduate Teaching Assistantships

The department awards many upper-level undergraduates the opportunity to act as recitation instructors to our freshman courses, enabling them to practice the art of teaching and talking mathematics and to earn a valuable credential while studying for their degree.

Admission

Admission to the Ph.D. program is based on academic records, letters of recommendation, and Graduate Record Examination scores. International applicants are required to submit a TOEFL or IELTS score if English is not their native language.

Basic Program

Graduate study is centered around three core areas:

- **Analysis**
  - AS.110.605 Real Variables
  - AS.110.607 Complex Variables
  - AS.110.608 Riemann Surfaces
  - AS.110.631 Partial Differential Equations I
  - & AS.110.632 Partial Differential Equations II
  - AS.110.645 Riemannian Geometry
  - & AS.110.646 and Riemannian Geometry

- **Algebra**
  - AS.110.601 Algebra
  - & AS.110.602 and Algebra
  - AS.110.617 Number Theory
  - AS.110.619 Lie Groups and Lie Algebras
  - AS.110.643 Algebraic Geometry
  - & AS.110.644 and Algebraic Geometry

- **Topology**
  - AS.110.615 Algebraic Topology
  - & AS.110.616 and Algebraic Topology

These 600-level graduate courses are preliminary to research and are built upon the foundations constituted by the 400-level courses:

- AS.110.401 Advanced Algebra I
- AS.110.402 Advanced Algebra II
- AS.110.405 Analysis I
- AS.110.406 Analysis II
- AS.110.311 Methods of Complex Analysis
- AS.110.413 Introduction To Topology

The 700-level courses are designed to bring students abreast of recent developments and to prepare them for research in the area of their choice.

Requirements for the M.A. Degree

Although the Mathematics Department does not admit students seeking a terminal M.A. degree, students in the Ph.D. program may earn an M.A. degree. Advanced undergraduate students may also apply to be admitted to the accelerated B.A./M.A. program.

M.A. candidates must complete:

- Four graduate courses given by the Johns Hopkins Mathematics Department;
- Two additional courses at the graduate or 400-level, other than AS.110.401, AS.110.405, and AS.110.415, given by the Johns Hopkins Mathematics Department, or, with the permission of the graduate program director, graduate mathematics courses given by other departments or universities.

All courses used to satisfy the requirements must be completed with a grade of B- or better. (Advanced graduate courses completed with a grade of P can also be used to satisfy the requirements.)

Requirements for the Ph.D. Degree

The departmental requirements for the Ph.D. degree are:

1. Candidates must show satisfactory work in Algebra (AS.110.601-AS.110.602), Real Variables (AS.110.605), Complex Variables (AS.110.607), Algebraic Topology (AS.110.615), and one additional mathematics graduate course in their first year. The seminars and qualifying exam preparation course cannot be used to fulfill this requirement. The algebra and analysis requirements can be satisfied by passing the corresponding written qualifying exam in September of the first year; these students must complete at least two courses each semester. Students having sufficient background in topology can substitute an advanced topology course for AS.110.615, with the permission of the instructor.

2. Candidates must pass written qualifying exams by the beginning of their second year in Analysis (Real and Complex) and in Algebra. Exams are scheduled for September and May of each academic year.

3. Candidates must show satisfactory work in at least two mathematics graduate courses each semester of their second year, and, if they have not passed their oral qualifying exam, in the first semester of their third year.

4. Candidates must pass a departmental oral qualifying examination in the student’s chosen area of research by April 8th of the third year. The topic of the exam is chosen in consultation with the faculty member who has agreed (provisionally) to be the student’s thesis advisor, who will also be involved in administering the exam.

5. There is no longer a Mathematics Department foreign language requirement. With the vast majority of articles written in English, the importance of having the capability of reading another language has diminished. However, important earlier literature in certain areas of mathematics may be written in French, German, or Russian. Moreover, some articles are still being written in French. It is now at the discretion of the student’s thesis advisor whether to impose a language requirement.

6. Candidates must produce a dissertation based upon independent and original research.

7. Candidates will gain teaching experience in mathematics as a teaching assistant for undergraduate courses. The student will be under the supervision of both the faculty member teaching the course and the director of undergraduate studies. First year students are given a reduced TA workload in the spring semester (this is related to item #2).

8. After completion of the thesis research the student will defend their dissertation by means of the Graduate Board Oral Exam. The exam must be held at least three weeks before the Graduate Board deadline the candidate wishes to meet.

Financial Aid

Students admitted to the Ph.D. program receive teaching assistantships and full tuition fellowships. Exceptional applicants become candidates for one of the university’s George E. Owen Fellowships.
**William Kelso Morrill Award**

The William Kelso Morrill Award for excellence in the teaching of mathematics is awarded every spring to the graduate student who best exemplifies the traits of Kelso Morrill: a love of mathematics, a love of teaching, and a concern for students.

**Excellence in Teaching Awards**

Three awards are given each year to a junior faculty member and graduate student teaching assistants who have demonstrated exceptional ability and commitment to undergraduate education.

For current faculty and contact information go to [http://www.mathematics.jhu.edu/new/people/people-faculty.htm](http://www.mathematics.jhu.edu/new/people/people-faculty.htm).

**Faculty**

**Chair**

Nitu Kitchloo
Professor; Symplectic geometry, topology of Kac-Moody groups, classical algebraic topology.

**Professors**

Caterina Consani
Arithmetic geometry, number theory, and non-commutative geometry.

Hans Lindblad
Harmonic analysis, PDE, fluid dynamics, relativity.

Chikako Mese
Geometric analysis.

Mauro Maggioni

Jack Morava
Algebraic topology, mathematical physics.

Kate Okikiolu
Harmonic analysis, spectral theory, geometry.

David Savitt
Number theory, Galois representations.

Bernard Shiffman
Several complex variables, complex geometry.

Vyacheslav V. Shokurov
Algebraic geometry.

Yannick Sire
Harmonic analysis, real geometry, complex geometry.

Christopher Sogge
J.J. Sylvester Professor; Fourier analysis, partial differential equations.

Joel Spruck
J.J. Sylvester Professor; Partial differential equations, geometric analysis.

W. Stephen Wilson
Algebraic topology, homotopy theory.

**Assistant Professors**

Jacob Bernstein
Minimal surface theory, mean curvature flow.

Benjamin Dodson
Partial differential equations, harmonic analysis.

Emily Riehl
Homotopy theory.

Brian Smithling
Arithmetic, algebraic geometry.

Yi Wang
Geometric Analysis, PDE, Harmonic Analysis, Conformed Geometry.

**Associate Teaching Professor**

Richard Brown
Director of Undergraduate Studies; Dynamical systems, low-dimensional topology.

**Emeriti**

J. Michael Boardman
Differential topology, algebraic topology.

Philip Hartman
Differential equations, differential geometry.

Takashi Ono
Algebra, number theory, algebraic groups.

**J.J. Sylvester Assistant Professor**

Giovanni DiMatteo
Number theory.

Jonas Luehrmann
Partial differential equations, mathematical physics.

Mona Merling
Homotopy Theory.

Hang Xu
Complex geometry.

YingYing Zhang
Geometric analysis, complex geometry.

Xudong Zhang
Algebraic geometry.

**Associate Research Scientist/Lecturer**

Jian Kong
IT Senior Lecturer; Algebraic geometry.

**Assistant Research Scientist and Senior Lecturer**

Wenjing Liao
High-dimensional data analysis, machine learning, imaging, signal processing.

James Murphy
Harmonic analysis, machine learning, remotely sensed image processing.

**Assistant Research Professor**

Sui Tang
Applied harmonic analysis, mathematical signal processing.

Stefano Vigogna
Courses

AS.110.105. Introduction to Calculus. 4.00 Credits.
This course starts from scratch and provides students with all the background necessary for the study of calculus. It includes a review of algebra, trigonometry, exponential and logarithmic functions, coordinates and graphs. Each of these tools will be introduced in its cultural and historical context. The concept of the rate of change of a function will be introduced. Not open to students who have studied calculus in high school.
Instructor(s): M. Martin
Area: Quantitative and Mathematical Sciences.

AS.110.106. Calculus I (Biology and Social Sciences). 4.00 Credits.
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. Many applications to the biological and social sciences will be discussed.
Instructor(s): S. Vigogna
Area: Quantitative and Mathematical Sciences.

AS.110.107. Calculus II (For Biological and Social Science). 4.00 Credits.
Differential and integral Calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. Applications to the biological and social sciences will be discussed, and the courses are designed to meet the needs of students in these disciplines.
Instructor(s): S. Vigogna, V. Lorman
Area: Quantitative and Mathematical Sciences.

AS.110.108. Calculus I. 4.00 Credits.
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor’s theorem and applications, infinite sequences and series. Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines.
Instructor(s): H. Xu
Area: Quantitative and Mathematical Sciences.

AS.110.109. Calculus II (For Physical Sciences and Engineering). 4.00 Credits.
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor’s theorem and applications, infinite sequences and series. Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines.
Instructor(s): M. Arap; X. Zheng
Area: Quantitative and Mathematical Sciences.

AS.110.113. Honors Single Variable Calculus. 4.00 Credits.
This is an honors alternative to the Calculus sequences AS.110.106-AS.110.107 or AS.110.108-AS.110.109 and meets the general requirement for both Calculus I and Calculus II (although the credit hours count for only one course). It is a more theoretical treatment of one variable differential and integral calculus and is based on our modern understanding of the real number system as explained by Cantor, Dedekind, and Weierstrass. Students who want to know the why’s and how’s of Calculus will find this course rewarding. Previous background in Calculus is not assumed. Students will learn differential Calculus (derivatives, differentiation, chain rule, optimization, related rates, etc), the theory of integration, the fundamental theorem(s) of Calculus, applications of integration, and Taylor series. Students should have a strong ability to learn mathematics quickly and on a higher level than that of the regular Calculus sequences.
Instructor(s): V. Lorman
Area: Quantitative and Mathematical Sciences.

AS.110.160. Discover Hopkins: Mathematics of Infinity. 1.00 Credit.
An interdisciplinary introduction to the history of infinity in mathematics, from Zeno’s paradox to the development of calculus to the crisis in the foundations of mathematics in the early 20th century. We will read about history, discuss philosophy, and learn some mathematics (including a crash course in mathematical logic and proof, building up to the rigorous definition of limits). A previous course in calculus is not required, but some mathematical maturity will be necessary.
Instructor(s): V. Lorman
Area: Quantitative and Mathematical Sciences.

AS.110.201. Linear Algebra. 4.00 Credits.
Prerequisites: Grade of C- or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam.
Instructor(s): G. Di Matteo; J. Bernstein
Area: Quantitative and Mathematical Sciences.

AS.110.202. Calculus III. 4.00 Credits.
Calculus of functions of more than one variable: partial derivatives, and applications; multiple integrals, line and surface integrals; Green’s Theorem, Stokes’ Theorem, and Gauss’ Divergence Theorem.
Prerequisites: Grade of C- or better in AS.110.107 OR AS.110.109 OR AS.110.113, or a 5 on the AP BC exam.
Instructor(s): G. Di Matteo; V. Pingali
Area: Quantitative and Mathematical Sciences.

AS.110.211. Honors Multivariable Calculus. 4.00 Credits.
This course includes the material in AS.110.202 with some additional applications and theory. Recommended for mathematically able students majoring in physical science, engineering, or especially mathematics.
AS.110.211-AS.110.212 used to be an integrated yearlong course, but now the two are independent courses and can be taken in either order.
Prerequisites: Pre/Co-Requisite: 110.201 or 110.212
Instructor(s): Y. Zhang
Area: Quantitative and Mathematical Sciences.
AS.110.212. Honors Linear Algebra. 4.00 Credits.
This course includes the material in AS.110.201 with some additional applications and theory. Recommended for mathematically able students majoring in physical science, engineering, or mathematics. AS.110.211-AS.110.212 used to be an integrated yearlong course, but now the two are independent courses and can be taken in either order. This course satisfies a requirement for the math major that its non-honors sibling does not.

**Prerequisites:** Grade of B+ or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam.

Instructor(s): J. Gell-redman
Area: Quantitative and Mathematical Sciences.

AS.110.225. Problem Solving Lab. 2.00 Credits.
An introduction to some fundamental techniques of mathematical problem solving, such as induction, invariants, inequalities and generating functions.

Instructor(s): D. Savitt
Area: Quantitative and Mathematical Sciences.

AS.110.302. Diff Equations/Applic. 4.00 Credits.
This is an applied course in ordinary differential equations, which is primarily for students in the biological, physical and social sciences, and engineering. The purpose of the course is to familiarize the student with the techniques of solving ordinary differential equations. The specific subjects to be covered include first order differential equations, second order linear differential equations, applications to electric circuits, oscillation of solutions, power series solutions, systems of linear differential equations, autonomous systems, Laplace transforms and linear differential equations, mathematical models (e.g., in the sciences or economics).

**Prerequisites:** Grade of C- or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam.

Instructor(s): R. Brown; Y. Sire
Area: Quantitative and Mathematical Sciences.

AS.110.304. Elementary Number Theory. 4.00 Credits.
The student is provided with many historical examples of topics, each of which serves as an illustration of and provides a background for many years of current research in number theory. Primes and prime factorization, congruences, Euler’s function, quadratic reciprocity, primitive roots, solutions to polynomial congruences (Chevalley’s theorem), Diophantine equations including the Pythagorean and Pell equations, Gaussian integers, Dirichlet’s theorem on primes.

**Prerequisites:** Grade of C- or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam.

Instructor(s): T. Wright; W. Wilson
Area: Quantitative and Mathematical Sciences.

AS.110.306. Honors Differential Equations. 4.00 Credits.
This course includes the material in 110.302 Differential Equations but with a strong emphasis on theory and proofs. Recommended only for mathematics majors or mathematically able students majoring in physical science or engineering.

**Prerequisites:** Grade of B+ or better in AS.110.107 or AS.110.109 or AS.110.113 or a 5 on the Advanced Placement BC exam.

Instructor(s): J. Gell-redman
Area: Quantitative and Mathematical Sciences.

AS.110.311. Methods of Complex Analysis. 4.00 Credits.
This course is an introduction to the theory of functions of one complex variable. Its emphasis is on techniques and applications, and it serves as a basis for more advanced courses. Functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions.

**Prerequisites:** Grade of C- or better in 110.202 or 110.211

Instructor(s): Y. Sire
Area: Quantitative and Mathematical Sciences.

AS.110.321. Honors Complex Analysis. 4.00 Credits.
This course is an introduction to the theory of functions of one complex variable for honors students. Its emphasis is on techniques and applications, and it serves as a basis for more advanced courses. Topics will include functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions, as well as applications to number theory and harmonic analysis.

Instructor(s): B. Dodson
Area: Quantitative and Mathematical Sciences.

AS.110.328. Non-Euclidean Geometry. 4.00 Credits.
For 2,000 years, Euclidean geometry was the geometry. In the 19th century, new, equally consistent but very different geometries were discovered. This course will delve into these geometries on an elementary but mathematically rigorous level.

Instructor(s): M. Merling
Area: Quantitative and Mathematical Sciences.

AS.110.401. Advanced Algebra I. 4.00 Credits.
An introduction to the basic notions of modern algebra. Elements of group theory: groups, subgroups, normal subgroups, quotients, homomorphisms. Generators and relations, free groups, products, commutative (Abelian) groups, finite groups. Groups acting on sets, the Sylow theorems. Definition and examples of rings and ideals. Introduction to field theory. Linear algebra over a field. Field extensions, constructible polygons, non-trisectability.

**Prerequisites:** Grade of C- or better in 110.201 or 110.212

Instructor(s): J. Kong
Area: Quantitative and Mathematical Sciences.

AS.110.402. Advanced Algebra II. 4.00 Credits.
Splitting field of a polynomial, algebraic closure of a field. Galois theory: correspondence between subgroups and subfields. Solvability of polynomial equations by radicals.

Instructor(s): B. Smithling
Area: Quantitative and Mathematical Sciences.

AS.110.405. Analysis I. 4.00 Credits.
This course is designed to give a firm grounding in the basic tools of analysis. It is recommended as preparation (but may not be a prerequisite) for other advanced analysis courses. Real and complex number systems, topology of metric spaces, limits, continuity, infinite sequences and series, differentiation, Riemann-Stieltjes integration.

**Prerequisites:** Grade of C- or better in 110.201 or 110.212 and 110.202 or 110.211

Instructor(s): J. Luehrmann
Area: Quantitative and Mathematical Sciences.
AS.110.406. Analysis II. 4.00 Credits.
This course continues AS.110.405 with an emphasis on the fundamental notions of modern analysis. Sequences and series of functions, Fourier series, equicontinuity and the Arzela-Ascoli theorem, the Stone-Weierstrass theorem, functions of several variables, the inverse and implicit function theorems, introduction to the Lebesgue integral.
Instructor(s): J. Spruck
Area: Quantitative and Mathematical Sciences.

AS.110.413. Introduction To Topology. 4.00 Credits.
Topological spaces, connectedness, compactness, quotient spaces, metric spaces, function spaces. An introduction to algebraic topology: covering spaces, the fundamental group, and other topics as time permits.
Prerequisites: Grade of C- or better in 110.202 or 110.211
Instructor(s): W. Wilson
Area: Quantitative and Mathematical Sciences.

AS.110.415. Honors Analysis I. 4.00 Credits.
This highly theoretical sequence in analysis is reserved for the most able students. The sequence covers the real number system, metric spaces, basic functional analysis, the Lebesgue integral, and other topics.
Instructor(s): Y. Sire
Area: Quantitative and Mathematical Sciences.

AS.110.416. Honors Analysis II. 4.00 Credits.
Prerequisites: Grade of C- or better in AS.110.415
Instructor(s): B. Shiffman
Area: Quantitative and Mathematical Sciences.

AS.110.417. Partial Diff Equations. 4.00 Credits.
Prerequisites: Grade of C- or better in AS.110.202 or 110.211
Instructor(s): J. Spruck
Area: Quantitative and Mathematical Sciences.

AS.110.421. Dynamical Systems. 4.00 Credits.
This is a course in the modern theory of Dynamical Systems. Topic include existence and uniqueness of general ODEs, nonlinear analysis and stability, including bifurcation theory and stable and center manifolds, smooth flows, limit sets, Hamiltonian mechanics, perturbation theory and structural stability.
Prerequisites: Grade of C- or better in AS.110.201 or AS.110.212 OR AS.110.202 or AS.110.211 and 110.302
Instructor(s): H. Xu
Area: Quantitative and Mathematical Sciences.

AS.110.422. Representation Theory. 4.00 Credits.
This course will focus on the basic theory of representations of finite groups in characteristic zero: Schur's Lemma, Mashcke's Theorem and complete reducibility, character tables and orthogonality, direct sums and tensor products. The main examples we will try to understand are the representation theory of the symmetric group and the general linear group over a finite field. If time permits, the theory of Brauer characters and modular representations will be introduced.
Prerequisites: Prereq: (AS.110.201 OR AS.110.212 ) AND AS.110.401
Instructor(s): M. Merling
Area: Quantitative and Mathematical Sciences.

AS.110.423. Lie Groups for Undergraduates. 4.00 Credits.
This course is an introduction to Lie Groups and their representations at the upper undergraduate level. It will cover basic Lie Groups such as SU (2), U(n), the Euclidean Motion Group and Lorentz Group. This course is useful for students who want a working knowledge of group representations. Some aspects of the role of symmetry groups in particle physics such as some of the formal aspects of the electroweak and the strong interactions will also be discussed. Recommended Course Background: AS.110.202; prior knowledge of group theory (AS.110.401) would be helpful.
Instructor(s): S. Zucker
Area: Quantitative and Mathematical Sciences.

AS.110.427. Introduction Calculus of Variations. 4.00 Credits.
The calculus of variations is concerned with finding optimal solutions (shapes, functions, etc.) where optimality is measured by minimizing a functional (usually an integral involving the unknown functions) possibly with constraints. Applications include mostly one-dimensional (often geometric) problems: brachistochrone, geodesics, minimum surface area of revolution, isoperimetric problem, curvature flows, and some differential geometry of curves and surfaces. Recommended Course Background: Calculus III
Prerequisites: Grade of B+ or better in AS.110.201 and AS.110.202.
Instructor(s): Y. Zhang
Area: Quantitative and Mathematical Sciences.

AS.110.431. Knot Theory. 4.00 Credits.
The theory of knots and links is a facet of modern topology. The course will be mostly self-contained, but a good working knowledge of groups will be helpful. Topics include braids, knots and links, the fundamental group of a knot or link complement, spanning surfaces, and low dimensional homology groups.
Instructor(s): C. McTague
Area: Quantitative and Mathematical Sciences.

AS.110.439. Introduction To Differential Geometry. 4.00 Credits.
Theory of curves and surfaces in Euclidean space: Frenet equations, fundamental forms, curvatures of a surface, theorems of Gauss and Mainardi-Codazzi, curves on a surface; introduction to tensor analysis and Riemannian geometry; theorema egregium; elementary global theorems.
Prerequisites: Grade of C- or better in (AS.110.201 or AS.110.212) and (AS.110.202 or AS.110.211)
Instructor(s): J. Spruck
Area: Quantitative and Mathematical Sciences.
AS.110.441. Calculus on Manifolds. 4.00 Credits.
This course provides the tools for classical three-dimensional physics and mechanics. This course extends these techniques to the general locally Euclidean spaces (manifolds) needed for an understanding of such things as Maxwell's equations or optimization in higher dimensional contexts, e.g. in economics. The course will cover the theory of differential forms and integration. Specific topics include Maxwell's equations in terms of 4D Lorentz geometry, vector (in particular, tangent) bundles, an introduction to de Rham theory, and Sard's theorem on the density of regular values of smooth functions. The course is intended to be useful to mathematics students interested in analysis, differential geometry, and topology, as well as to students in physics and economics.
Instructor(s): J. Morava
Area: Quantitative and Mathematical Sciences.

AS.110.443. Fourier Analysis. 4.00 Credits.
Prerequisites: Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)
Instructor(s): Y. Wang
Area: Quantitative and Mathematical Sciences.

AS.110.446. Introduction to Statistical Learning, Data Analysis and Signal Processing. 4.00 Credits.
Introduction to high dimensional data sets: key problems in statistical and machine learning. Geometric aspects. Principal component analysis, linear dimension reduction, random projections. Concentration phenomena: examples and basic inequalities. Metric spaces and embeddings thereof. Kernel methods. Nonlinear dimension reduction, manifold models. Regression. Vector spaces of functions, linear operators, projections. Orthonormal bases; Fourier and wavelet bases, and their use in signal processing and time series analysis. Basic approximation theory. Linear models, least squares. Bias and variance tradeoffs, regularization. Sparsity and compressed sensing. Multiscale methods. Graphs and networks. Random walks on graphs, diffusions, page rank. Block models. Spectral clustering, classification, semi-supervised learning. Algorithmic and computational aspects of the above will be consistently in focus, as will be computational experiments on synthetic and real data. Linear algebra will be used throughout the course, as will multivariable calculus and basic probability (discrete random variables). Basic experience in programming in C or MATLAB or R or Octave. Recommended Course Background: More than basic programming experience in Matlab or R; some more advanced probability (e.g. continuous random variables), some signal processing (e.g. Fourier transform, discrete and continuous).
Co-listed with EN.550.416
Prerequisites: AS.110.201
Instructor(s): M. Maggioni
Area: Quantitative and Mathematical Sciences.

AS.110.503. Undergraduate Research in Mathematics. 0.00 - 4.00 Credits.
Instructor(s): C. Mese; C. Sogge; S. Raymond; S. Zucker.

AS.110.586. Independent Study. 0.00 - 4.00 Credits.
Instructor(s): Staff.

AS.110.595. Internship. 1.00 Credit.

AS.110.601. Algebra.
An introductory graduate course on fundamental topics in algebra to provide the student with the foundations for number theory, algebraic geometry, and other advanced courses. Topics include group theory, commutative algebra, Noetherian rings, local rings, modules, rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras.
Instructor(s): V. Shokurov
Area: Quantitative and Mathematical Sciences.

AS.110.602. Algebra.
An introductory graduate course on fundamental topics in algebra to provide the student with the foundations for Number Theory, Algebraic Geometry, and other advanced courses. Topics include group theory, commutative algebra, Noetherian rings, local rings, modules, and rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras. Recommended Course Background: AS.110.401-AS.110.402
Instructor(s): S. Zucker
Area: Quantitative and Mathematical Sciences.

AS.110.605. Real Variables.
Measure and integration on abstract and locally compact spaces (extension of measures, decompositions of measures, product measures, the Lebesgue integral, differentiation, Lp-spaces); introduction to functional analysis; integration on groups; Fourier transforms.
Instructor(s): C. Mese
Area: Quantitative and Mathematical Sciences.

AS.110.607. Complex Variables.
Analytic functions of one complex variable. Topics include Mittag-Leffler Theorem, Weierstrass factorization theorem, elliptic functions, Riemann-Roch theorem, Picard theorem, and Nevanlinna theory. Recommended Course Background: AS.110.311, AS.110.405
Instructor(s): C. Mese.

AS.110.608. Riemann Surfaces.
Abstract Riemann surfaces. Examples: algebraic curves, elliptic curves and functions on them. Holomorphic and meromorphic functions and differential forms, divisors and the Mittag-Leffler problem. The analytic genus. Bezout's theory and applications. Introduction to sheaf theory, with applications to constructing linear series of meromorphic functions. Serre duality, the existence of meromorphic functions on Riemann surfaces, the equality of the topological and analytic genera, the equivalence of algebraic curves and compact Riemann surfaces, the Riemann-Roch theorem. Period matrices and the Abel-Jacobi mapping, Jacobi inversion, the Torelli theorem. Uniformization (time permitting).
Instructor(s): B. Shiffman; C. Mese.

AS.110.612. Complex Geometry.
Instructor(s): B. Shiffman.

AS.110.615. Algebraic Topology.
Polyhedra, simplicial and singular homology theory, Lefschetz fixed-point theorem, cohomology and products, homological algebra, Künneth and universal coefficient theorems, Poincaré and Alexander duality theorems.
Instructor(s): C. Mese; J. Morava.
AS.110.616. Algebraic Topology.
Polyhedra, simplicial and singular homology theory, Lefschetz fixed-point theorem, cohomology and products, homological algebra, Kunneth and universal coefficient theorems, Poincaré; and Alexander duality theorems.
Prerequisites: AS.110.401 AND AS.110.413
Instructor(s): M. Merling
Area: Quantitative and Mathematical Sciences.

AS.110.617. Number Theory.
Topics in advanced algebra and number theory, including local fields and adeles, Iwasawa-Tate theory of zeta functions and connections with Hecke's treatment, semisimple algebras over local and number fields, adeles geometry.
Instructor(s): C. Mese; G. Di Matteo
Area: Quantitative and Mathematical Sciences.

AS.110.618. Number Theory.
Topics in advanced algebra and number theory, including local fields and adeles, Iwasawa-Tate theory of zeta-functions and connections with Hecke's treatment, semi-simple algebras over local and number fields, adeles geometry.
Instructor(s): D. Savitt.

Lie groups and Lie algebras, classification of complex semi-simple Lie algebras, compact forms, representations and Weyl formulas, symmetric Riemannian spaces.
Prerequisites: AS.110.402
Instructor(s): C. Mese; J. Morava
Area: Quantitative and Mathematical Sciences.

AS.110.631. Partial Differential Equations I.
An introductory graduate course in partial differential equations. Classical topics include first order equations and characteristics, the Cauchy-Kowalewski theorem, Laplace's equations, heat equation, wave equation, fundamental solutions, weak solutions, Sobolev spaces, maximum principles.
Prerequisites: AS.110.605
Instructor(s): B. Dodson.

AS.110.632. Partial Differential Equations II.
An introductory graduate course in partial differential equations. Classical topics include first order equations and characteristics, the Cauchy-Kowalevski theorem, Laplace's equation, heat equation, wave equation, fundamental solutions, weak solutions, Sobolev spaces, maximum principles. The second term focuses on special topics such as second order elliptic theory.
Instructor(s): C. Mese; H. Lindblad; R. Brown.

AS.110.633. Harmonic Analysis.
Fourier multipliers, oscillatory integrals, restriction theorems, Fourier integral operators, pseudodifferential operators, eigenfunctions. Undergrads need instructor's permission.
Instructor(s): C. Sogge
Area: Quantitative and Mathematical Sciences.

AS.110.634. Riemannian Geometry.
Differential manifolds, vector fields, flows, Frobenius' theorem. Differential forms, deRham's theorem, vector bundles, connections, curvature, Chern classes, Cartan structure equations. Riemannian manifolds, Bianchi identities, geodesics, exponential maps. Geometry of submanifolds, hypersurfaces in Euclidean space. Other topics as time permits, e.g., harmonic forms and Hodge theorem, Jacobi equation, variation of arc length and area, Chern-Gauss-Bonnet theorems.
Instructor(s): B. Shiffman; C. Mese.

AS.110.635. Microlocal Analysis.
Microlocal analysis is the geometric study of singularities of solutions of partial differential equations. The course will begin by introducing the geometric theory of (Schwartz) distributions: Fourier transform and Sobolev spaces, pseudo-differential operators, wave front set of a distribution, elliptic operators, Lagrangean distributions, oscillatory integrals, method of stationary phase, Fourier integral operators. The second semester will develop the theory and apply it to special topics such as asymptotics of eigenvalues/eigenfunctions of the Laplace operator on a Riemann manifold, linear and non-linear wave equation asymptotics of quantum systems, Bochner-Riesz means, maximal theorems.
Instructor(s): H. Lindblad.

AS.110.636. Functional Analysis.
Instructor(s): C. Mese; J. Spruck.

AS.110.641. Algebraic Geometry.
Affine varieties and commutative algebra. Hilbert's theorems about polynomials in several variables with their connections to geometry. General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, connections with complex analytic geometry and topology, schemes.
Instructor(s): V. Shokurov.

AS.110.642. Algebraic Geometry.
Affine varieties and commutative algebra. Hilbert's theorems about polynomials in several variables with their connections to geometry. General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch Theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, schemes, connections with complex analytic geometry and topology.
Instructor(s): C. Consani
Area: Quantitative and Mathematical Sciences.

AS.110.643. Algebraic Geometry.
General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch Theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, schemes, connections with complex analytic geometry and topology.
Instructor(s): C. Consani
Area: Quantitative and Mathematical Sciences.

AS.110.644. Algebraic Geometry.
General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch Theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, schemes, connections with complex analytic geometry and topology.
Instructor(s): C. Consani
Area: Quantitative and Mathematical Sciences.

AS.110.645. Riemannian Geometry.
Differential manifolds, vector fields, flows, Frobenius’ theorem. Differential forms, deRham’s theorem, vector bundles, connections, curvature, Chern classes, Cartan structure equations. Riemannian manifolds, Bianchi identities, geodesics, exponential maps. Geometry of submanifolds, hypersurfaces in Euclidean space. Other topics as time permits, e.g., harmonic forms and Hodge theorem, Jacobi equation, variation of arc length and area, Chern-Gauss-Bonnet theorems.
Instructor(s): B. Shiffman; C. Mese.

AS.110.646. Riemannian Geometry.
The goal is to give a self-contained course on mean curvature flow, starting with the basic linear heat equation in Euclidean space and – hopefully – getting to topics of current research. Mean curvature flow is a geometric heat equation that shares many properties with Ricci flow, harmonic map heat flow, Yang-Mills flow and the Navier-Stokes equations. Recommended Course Background: AS.110.605 and an undergraduate course in differential geometry; AS.110.645 and AS.110.631
Instructor(s): C. Mese.

AS.110.712. Topics in Mathematical Physics.
Instructor(s): J. Morava.
AS.110.722. Topics in Homotopy Theory.
The course will focus on recent developments in homotopy theory, such as Galois theory for \( E_n \) \((n \geq 2)\) ring-spectra, and on connections with number theory; in particular, work of Bhatt, Hesselholt, Lurie, Scholze and others on topological Hochschild homology and its applications to geometry over the \( p \)-adic complex numbers.
Instructor(s): J. Morava
Area: Quantitative and Mathematical Sciences.

AS.110.748. Topics in Geometry.
Instructor(s): B. Shiffman
Area: Quantitative and Mathematical Sciences.

AS.110.722. Topics in Arithmetic Geometry.
Topics around the subject of Arithmetic Geometry will be covered in this course.
Instructor(s): B. Smithling
Area: Quantitative and Mathematical Sciences.

AS.110.726. Topics in Analysis.
The topics covered will involve the theory of calculus of Functors applied to Geometric problems like Embedding theory. Other related areas will be covered depending on the interest of the audience.
Instructor(s): Y. Sire
Area: Quantitative and Mathematical Sciences.

AS.110.727. Topics/Algebraic Topology.
Instructor(s): C. Mese; E. Riehl.

AS.110.728. Topics in Algebraic Topology.
Instructor(s): N. Kitchloo.

AS.110.731. Topics in Geometric Analysis.
Instructor(s): Y. Wang.

AS.110.735. Topics in Analysis.
Instructor(s): B. Smithling.

AS.110.737. Topics in Algebraic Geometry.
Instructor(s): C. Consani.

AS.110.738. Topics in Algebraic Topology.
Introduction to toric varieties. This class is a general introduction to toric varieties. Toric varieties are special kinds of algebraic varieties which can be described by lattices and convex sets. They provide a rich source of concrete examples in complex geometry or mathematical physics. If time permits, we discuss in the end the stability of toric embeddings. Students should know basic notions of algebraic geometry (schemes, sheaves, linear systems), as covered in AS.110.643.
Instructor(s): C. Mese, R. Brown; V. Shokurov
Area: Quantitative and Mathematical Sciences.

AS.110.741. Topics in Partial Differential Equations.
Instructor(s): H. Lindblad.

AS.110.742. Topics in Partial Differential Equations.
In this course we will be discussing some dispersive evolution equations, primarily the nonlinear Schrodinger equation. Topics will include well-posedness theory, conservation laws, and scattering. The course will be accessible to students who have not taken graduate partial differential equations or functional analysis.
Instructor(s): B. Dodson.

AS.110.749. Topics in Differential Geometry.
In this class, we will study Aaron Naber and Jeff Cheeger’s recent result on proving codimension four conjecture. We plan to talk about some early results of the structure on manifolds with lower Ricci bound by Cheeger and Colding. We will prove quantitative splitting theorem, volume convergence theorem, and the result that almost volume cone implies almost metric cone. Then we will discuss regularity of Einstein manifolds and the codimension four conjecture.
Instructor(s): Y. Wang
Area: Quantitative and Mathematical Sciences.

AS.110.755. Topics in Fluid Dynamics.
Graduate students only.
Instructor(s): J. Bernstein
Area: Quantitative and Mathematical Sciences.

AS.110.756. Topics in Algebra.
This will be a course in commutative algebra. Topics may include: Noetherian rings and modules, the Nullstellensatz, Hilbert basis theorem, localization, integrality, Noether normalization, primary decomposition, DVRs, Dedekind domains, dimension theory, smoothness and regularity, and homological methods.
Instructor(s): C. Consani
Area: Quantitative and Mathematical Sciences.

AS.110.758. Topics in Complex Geometry.
Instructor(s): B. Shiffman.

AS.110.761. Topics in Topology.
Instructor(s): N. Kitchloo
Area: Quantitative and Mathematical Sciences.

AS.110.762. Topics in Topology II.
Instructor(s): N. Kitchloo.

AS.110.764. Topics in Riemannian Geometry.
Topics courses are restricted to graduate students only.
Instructor(s): J. Bernstein.

AS.110.790. Seminar in Complex Geometry.
Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.
Instructor(s): V. Pingali
Area: Quantitative and Mathematical Sciences.

Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.
Instructor(s): C. Sogge
Area: Quantitative and Mathematical Sciences.

AS.110.792. Seminar in Algebraic Geometry and Number Theory.
Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.
Instructor(s): B. Smithling
Area: Quantitative and Mathematical Sciences.

AS.110.793. Seminar in Topology.
For graduate students only. Presentations of current research papers by faculty, graduate students and invited guest speakers.
Instructor(s): W. Wilson
Area: Quantitative and Mathematical Sciences.

AS.110.795. Seminar in Data Analysis.
Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.
Instructor(s): M. Maggioni
Area: Quantitative and Mathematical Sciences.
AS.110.796. Seminar in Complex Analysis.
Seminar on current research in functions of several complex variables.
Graduate students only.
Instructor(s): B. Shiffman.

AS.110.797. Seminar in Chromatic Homotopy.
Undergraduates with instructors permission or signature from the
Program Coordinator.
Instructor(s): J. Morava.

AS.110.798. Seminar in Number Theory.
Presentations of current research papers by faculty, graduate students
and invited guest speakers. For graduate students only.
Instructor(s): B. Smithling; C. Consani.

AS.110.799. Seminar in Algebraic Geometry.
For graduate students only. Presentations of current research papers by
faculty, graduate students and invited guest speakers.
Instructor(s): J. Martinez Garcia; V. Shokurov.

AS.110.800. Independent Study-Graduates.
Area: Quantitative and Mathematical Sciences.

AS.110.801. Thesis Research.
Instructor(s): Staff.